

Cosmological Constraint and Analysis on Holographic Dark Energy Model Characterized by the Conformal-age-like Length

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We present a best-fit analysis on the single-parameter holographic dark energy model characterized by the conformal-age-like length, $L = \frac{1}{a^4(t)} \int_0^t dt' a^3(t')$. Based on the Union2 compilation of 557 supernova Ia data, the baryon acoustic oscillation results from the SDSS DR7 and the cosmic microwave background radiation data from the WMAP7, we show that the model gives the minimal $\chi_{min}^2 = 546.273$, which is comparable to $\chi_{\Lambda\text{CDM}}^2 = 544.616$ for the ΛCDM model. The single parameter d concerned in the model is found to be $d = 0.232 \pm 0.006 \pm 0.009$. Since the fractional density of dark energy $\Omega_{de} \sim d^2 a^2$ at $a \ll 1$, the fraction of dark energy is naturally negligible in the early universe, $\Omega_{de} \ll 1$ at $a \ll 1$. The resulting constraints on the present fractional energy density of matter and the equation of state are $\Omega_{m0} = 0.286_{-0.018}^{+0.019} {}_{-0.028}^{+0.032}$ and $w_{de0} = -1.240_{-0.027}^{+0.027} {}_{-0.044}^{+0.045}$ respectively. The model leads to a slightly larger fraction of matter comparing to the ΛCDM model. We also provide a systematic analysis on the cosmic evolutions of the fractional energy density of dark energy, the equation of state of dark energy, the deceleration parameter and the statefinder. It is noticed that the equation of state crosses from $w_{de} > -1$ to $w_{de} < -1$, the universe transits from decelerated expansion ($q > 0$) to accelerated expansion ($q < 0$) recently, and the statefinder may serve as a sensitive diagnostic to distinguish the CHDE model with the ΛCDM model.

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I. INTRODUCTION

Observations of Type Ia supernovae (SNIa) [1, 2], cosmic microwave background (CMB) [3] and large scale structure (LSS) [4] have complementarily established the present acceleration of the universe expansion. Within the framework of the general relativity, a consistent picture has indicated that nearly three quarters of our universe consists of a mysterious negative pressure component named by dark energy, which is responsible for the accelerated expansion. However, the nature of such an exotic energy component is still rather uncertain. The simplest candidate is a positive cosmological constant. Although fitting the observations well, a cosmological constant, however, is plagued with the fine-tuning problem and the coincidence problem [5].

The holographic principle [6, 7] indicates that the vacuum energy calculated in quantum field theory might take too many degrees of freedom into consideration which results in the fine-tuning problem of the cosmological constant. In [8], the author suggested that due to the limit set by the formation of a black hole, the ultraviolet (UV) cutoff Λ_{uv} in the effective field theory should be related to the infrared (IR) cutoff L , i.e., in terms of the natural units,

$$L^3 \Lambda_{\text{uv}}^4 \lesssim L M_p^2, \quad (1)$$

where M_p is the reduced Planck constant $M_p^2 = 1/(8\pi G)$ with G the Newton's constant. This means that the effective theory describes all states of the system, except those that have already collapsed to a black hole. Such a dramatic depletion of quantum states leads to much small vacuum energy density,

$$\rho_{\text{vac}} \sim \Lambda_{\text{uv}}^4 \sim M_p^2 L^{-2}. \quad (2)$$

If the IR cutoff L is comparable to the current Hubble radius, the resulting ρ_{vac} requires no magnificent cancelation to be consistent with observational bounds [8]. Many interesting studies on holography and cosmology are conducted [9–12]. In [13], we show that if the IR cutoff L is characterized with the total comoving horizon of the universe, the very large primordial part of the comoving horizon generated by the inflation of early universe might give some insights into the cosmological constant and the coincidence problem. Plenty of alternative models (for some reviews see [14–18]) have been proposed to provide the possible explanations for the recent cosmic acceleration. Based on the holographic principle [6–8], some interesting holographic dark energy models in which the dark energy density is assumed to scale as $\rho_{de} \propto M_p^2 L^{-2}$, were proposed and studied [19–41] by taking different choices of the characteristic length scale of the universe, L .

Especially, the age of the universe was chosen as the IR cutoff to build agegraphic dark energy model(ADE) [30]. In order to avoid some internal inconsistencies in the original model, a new version of this model (NADE) replacing the age of the universe by the conformal age of the universe [31]. An interesting comparison of some holographic dark energy models in [40] shows that the NADE seems not to be consistent with the cosmological observations very well. Writing the four dimensional spacetime volume at the cosmic time t as follows

$$\int d^3x \int_0^t dt' \sqrt{-g} = \left(a^3(t) \int d^3x \right) \cdot a(t) \cdot \left(\frac{1}{a^4(t)} \int_0^t dt' a^3(t') \right) \equiv V a(t) L, \quad (3)$$

where $V = a^3(t) \int d^3x$ is the physical space volume and $a(t)$ is the scale factor of the universe, and L is defined to be

$$L = \frac{1}{a^4(t)} \int_0^t dt' a^3(t') = \frac{1}{a^4(t)} \int \frac{dt'}{a(t')} a^4(t') \equiv \frac{1}{a^4(t)} \int d\eta' a^4(t') \quad (4)$$

which may be regarded as a conformal-age-like length scale of the universe. In [41], such conformal-age-like length is proposed to be the characteristic length scale of the universe to establish a holographic dark energy model (CHDE) which is similar to the new agegraphic dark energy model [31]. The conformal-age-like length $\frac{1}{a^4(t)} \int_0^t dt' a^3(t')$ is adopted rather than the age-like length $\frac{1}{a^3(t)} \int_0^t dt' a^3(t')$, this is because the model based on the age-like length seems to have the similar self-inconsistency to the agegraphic dark energy model(ADE)[30].

In this note, we are going to perform a best-fit analysis on the CHDE model by using the Union2 compilation of 557 supernova Ia (SNIa) data [42], the baryon acoustic oscillation (BAO) results from the Sloan Digital Sky Survey data release 7 (SDS DR7) [43] and the cosmic microwave background radiation (CMB) data from the 7-yr Wilkinson Microwave Anisotropy Probe (WMAP7) [44]. We find that the best-fit results of CHDE model are comparable to that of the Λ CDM model. Based on the observational constraints on the CHDE model, we also concentrate on the studies for the cosmic evolutions of some interesting quantities within the CHDE model. The paper is organized as follows: in Sec. II, we briefly introduce the CHDE model; in Sec. III, we carry out a best-fit analysis on the CHDE model; in Sec. IV, we study the cosmic evolutions of the fractional energy density of CHDE, the equation of state of CHDE, the deceleration parameter and statefinder; our concluding remarks are given in Sec. V.

II. BRIEF OUTLINE ON CHDE MODEL

Rewriting the conformal-age-like parameter as

$$L = \frac{1}{a^4(t)} \int_0^t dt' a^3(t') = \frac{1}{a^4(t)} \int_0^a a'^3 \frac{da'}{H' a'} , \quad (5)$$

with $H \equiv \dot{a}/a$ the Hubble parameter and “ \cdot ” denoting the derivative respect to cosmic time t , we then define the holographic dark energy with the density parameterized by the characteristic length scale L of the universe as follows

$$\rho_{de} = 3d^2 M_p^2 L^{-2} , \quad (6)$$

where d is a positive constant parameter. Correspondingly, the fractional energy density is given by the characteristic length scale L of the universe

$$\Omega_{de} = \frac{\rho_{de}}{3M_p^2 H^2} = \frac{d^2}{H^2 L^2} . \quad (7)$$

Considering a flat FRW universe containing matter, radiation and CHDE, we have the Friedmann equation

$$3M_p^2 H^2 = \rho_m + \rho_r + \rho_{de} . \quad (8)$$

When each component is conservative respectively, we get the equation for the density

$$\dot{\rho}_i + 3H(1 + w_i)\rho_i = 0 \quad (9)$$

with $i = m, r$ and de . Combining with Eqs.(5-7), the EoS of dark energy is given by [41]

$$w_{de} = -1 - \frac{8}{3} + \frac{2}{3d} \frac{\sqrt{\Omega_{de}}}{a} . \quad (10)$$

The conservations of matter and radiation result in $\rho_m = \rho_{m0} a^{-3} = \Omega_{m0} 3M_p^2 H_0^2 a^{-3}$ and $\rho_r = \rho_{r0} a^{-4} = \Omega_{r0} 3M_p^2 H_0^2 a^{-4}$ respectively, where the fractional energy densities are defined as $\Omega_i = \rho_i/\rho_c$ for $i = m$ and r , and $\rho_c = 3M_p^2 H^2$ is the critical energy density. Note that we have set the present scale factor of the universe $a_0 = 1$ and the subscript “0” always indicates the present values of the corresponding quantities. Thus the Friedmann equation Eq.(8) can be rewritten as

$$H^2 = \Omega_{m0} H_0^2 a^{-3} + \Omega_{r0} H_0^2 a^{-4} + \Omega_{de} H^2 . \quad (11)$$

Defining $r_0 = \Omega_{r0}/\Omega_{m0}$, we have

$$\Omega_r(a) = \frac{\rho_r}{3M_p^2 H^2} = \frac{r_0}{a + r_0} (1 - \Omega_{de}(a)) . \quad (12)$$

and

$$\Omega_m(a) = \frac{\rho_m}{3M_p^2 H^2} = \frac{a}{a + r_0} (1 - \Omega_{de}(a)) . \quad (13)$$

From the Friedmann equation Eq.(11), we have

$$\frac{1}{Ha} = \frac{1}{H_0 \sqrt{\Omega_{m0}}} \sqrt{a(1 - \Omega_{de})} \sqrt{\frac{1}{1 + r_0/a}} . \quad (14)$$

Referring to Eqs.(5) and (7), one gets

$$\int_0^a a'^3 \frac{da'}{H' a'} = \frac{a^5 d}{\sqrt{\Omega_{de}} Ha} . \quad (15)$$

Substituting Eq. (14) into above equation and taking derivative with respect to a in both sides, we obtain the differential equation of motion for Ω_{de} as [41]

$$\frac{d\Omega_{de}}{da} = \frac{\Omega_{de}}{a} (1 - \Omega_{de}) \left(11 + \frac{r_0}{a + r_0} - \frac{2}{d} \frac{\sqrt{\Omega_{de}}}{a} \right) . \quad (16)$$

Under the limit $1 - \Omega_{de} \simeq 1$ at $a \ll 1$, we arrive at the solution from the differential equation (16)

$$\Omega_{de} \simeq \frac{d^2}{4} \left(9 + \frac{r_0}{a + r_0} \right)^2 a^2 \quad (17)$$

which is a good approximation and consistent with the conditions $a \ll 1$ and $1 - \Omega_{de} \simeq 1$.

It is noticed that the above solution Eq.(17) reduces to the corresponding self-consistent approximation in radiation-dominated epoch ($r_0 \rightarrow 1$) and in matter-dominated epoch ($r_0 \rightarrow 0$) respectively [41]. Substituting Eq.(17) into Eq.(10), we yield the EoS of the dark energy in ambient-constituent-dominated epoch. Particularly, the EoS of dark energy is $w_{de} = -\frac{1}{3}$ in the radiation-dominated epoch, and then transits to $w_{de} = -\frac{2}{3}$ in matter-dominated epoch. Referring to Eq.(10), the EoS of dark energy w_{de} eventually turns to be $w_{de} < -1$ due to the expansion of the universe. Thus the CHDE is responsible of the present cosmic accelerated expansion.

It is more interesting to observe that with the analytic feature of the differential equation of motion for Ω_{de} , we are able to take the approximate solution Eq.(17) at certain point $a_i \ll 1$ as the initial condition to solve the differential equation of motion for Ω_{de} . Note that once d is given, the present fractional energy density $\Omega_{de}(a = 1)$ can be naturally obtained by solving Eq.(16), so the degrees of freedom of the CHDE model is the same as the one of the Λ CDM model.

Using the definition $a = 1/(1 + z)$ with z the redshift, we can rewrite Eq. (16) as

$$\frac{d\Omega_{de}}{dz} = -\frac{\Omega_{de}(1 - \Omega_{de})}{1 + z} \left(11 + \frac{r_0(1 + z)}{1 + r_0(1 + z)} - \frac{2}{d} \sqrt{\Omega_{de}}(1 + z) \right) . \quad (18)$$

Due to the analytical property mentioned above, we can take the approximate solution at $z_i \gg 1$ (or $a_i \ll 1$),

$$\Omega_{de}(z_i) = \frac{d^2}{4} \left(9 + \frac{r_0(1+z_i)}{1+r_0(1+z_i)} \right)^2 \frac{1}{(1+z_i)^2}, \quad (19)$$

as the initial condition to solve the differential equation of motion for Ω_{de} . The final solution depends weakly on the choice of z_i in a wide range as Ω_{de} is tiny and scales as $1/(1+z)^2$ at $z \gg 1$. This weak dependence is also checked directly by numerical method. In our numerical calculation, we simply set $z_i = 2000$.

III. BEST-FIT ANALYSIS ON CHDE MODEL

We will investigate the cosmological constraints on the CHDE model by using the Union2 compilation of 557 supernova Ia (SNIa) data [42], the baryon acoustic oscillation (BAO) results from the Sloan Digital Sky Survey data release 7 (SDS DR7) [43] and the cosmic microwave background radiation (CMB) data from the 7-yr Wilkinson Microwave Anisotropy Probe (WMAP7) [44]. The analysis method for the observational data are given in Appendix A. In the following, we give the expression of the important quantity used in best-fit analysis, $E(z) \equiv H(z)/H_0$.

From the Friedmann equation Eq.(11), we have

$$E(z) \equiv \frac{H(z)}{H_0} = \left(\frac{\Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4}{1 - \Omega_{de}(z)} \right)^{1/2}, \quad (20)$$

At $z < 2000$, the energy density of radiation is the sum of those of photons and relativistic neutrinos. Here $\Omega_{r0} = \Omega_{\gamma0} (1 + 0.2271N_{\text{eff}})$, where $\Omega_{\gamma0}$ is the present fractional photon energy density and $N_{\text{eff}} = 3.04$ is the effective number of neutrino species [44]. In this paper, we adopt the best-fit value, $\Omega_{\gamma0} = 2.469 \times 10^{-5} h^{-2}$ (for $T_{\text{CMB}} = 2.725$ K) with $h \equiv H_0/100/[\text{km sec}^{-1} \text{Mpc}^{-1}]$ given by WMAP7 [44]. For the given values of parameters d and r_0 , we can numerically solve $\Omega_{de}(z)$ from the differential equation Eq.(18) by using the initial condition Eq.(19). Substituting the result $\Omega_{de}(z = 0)$ into Eqs.(13) and (12), we obtain Ω_{r0} and Ω_{m0} . With all the results, we then yield the function $E(z)$ from Eq.(20).

The best-fit results are summarized in TABLE I. For comparison, we also give the fitting results for the Λ CDM model with the same observational data. Because of the tiny ratio of the fraction of radiation to that of matter $r_0 \sim O(10^{-4})$, we have $\Omega_{m0} + \Omega_{de0} \simeq 1$ in TABLE I. It is seen that the CHDE model favors slightly larger fraction of matter. Obviously, the present EoS of CHDE

TABLE I: The best-fit results constrained by the observational data at 1σ (68.3%) and 2σ (95.4%) confidence levels

| | χ^2 | d | $r_0 \times 10^4$ |
|---------------|----------|--|--|
| Λ CDM | 544.616 | -- | $3.057^{+0.100}_{-0.096} \quad ^{+0.165}_{-0.157}$ |
| CHDE | 546.273 | $0.232^{+0.006}_{-0.006} \quad ^{+0.009}_{-0.009}$ | $3.052^{+0.074}_{-0.073} \quad ^{+0.122}_{-0.120}$ |

| | Ω_{m0} | Ω_{de0} | h | w_{de0} (EOS) |
|---------------|--|--|--|---|
| Λ CDM | $0.277^{+0.021}_{-0.019} \quad ^{+0.035}_{-0.031}$ | $0.723^{+0.019}_{-0.021} \quad ^{+0.031}_{-0.035}$ | $0.702^{+0.016}_{-0.015} \quad ^{+0.026}_{-0.025}$ | -1 |
| CHDE | $0.286^{+0.019}_{-0.018} \quad ^{+0.032}_{-0.028}$ | $0.714^{+0.018}_{-0.019} \quad ^{+0.028}_{-0.032}$ | $0.692^{+0.016}_{-0.015} \quad ^{+0.026}_{-0.025}$ | $-1.240^{+0.027}_{-0.027} \quad ^{+0.045}_{-0.044}$ |

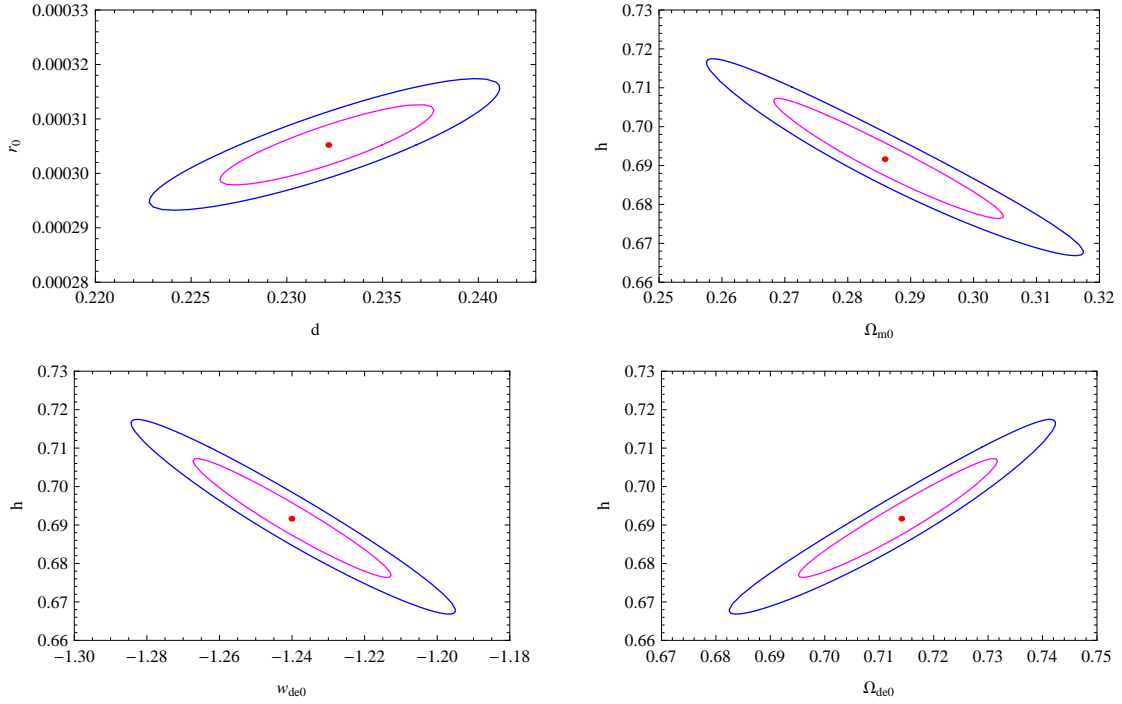


FIG. 1: Probability contours at 68.3% and 95.4% confidence levels for the CHDE model.

is below -1 significantly. In Fig. 1, we plot some probability contours at 68.3% and 95.4% confidence levels for the relevant cosmological quantities in the CHDE model.

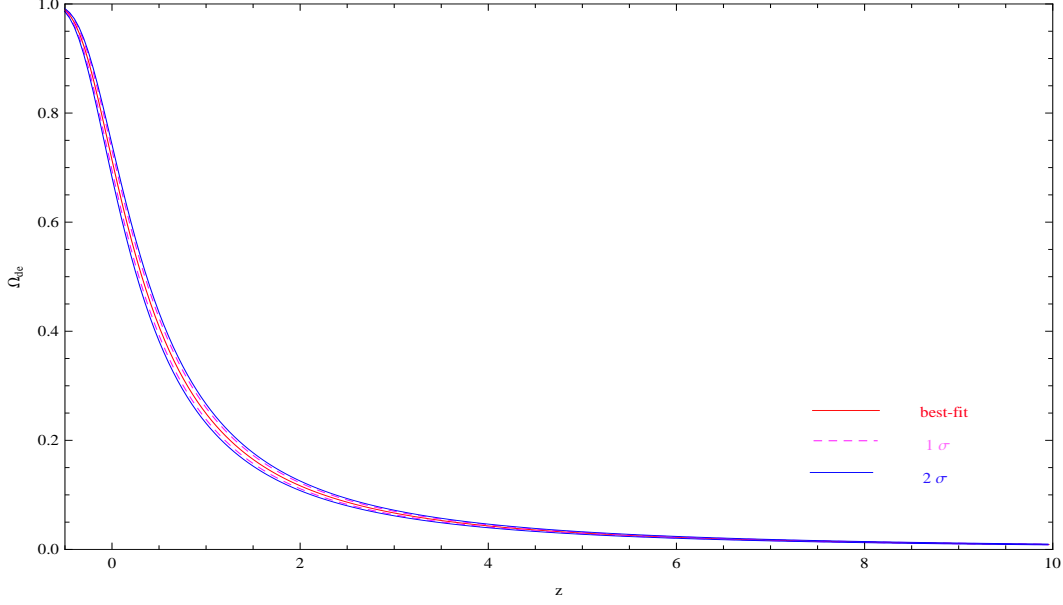


FIG. 2: Cosmic evolution of the fractional energy density of CHDE

IV. SYSTEMATIC STUDY ON CHDE MODEL

Taking the best-fit values of model parameters, we are able to investigate the cosmic evolutions of some interesting quantities by solving corresponding equations numerically. Here we are going to briefly study the cosmic evolutions of the fractional energy density Ω_{de} , the equation of state w_{de} , the deceleration parameter and statefinder [53, 54] in the CHDE model.

In Fig.2, we present the evolutionary trajectory of the fractional energy density of CHDE. It is seen that the fractional energy density of CHDE decreases rapidly with increase of the redshift and becomes tiny in the early universe, thus the model is consistent with primordial nucleosynthesis (BBN)[55]. Actually, this can be enlightened by the fact from Eq.(17) that $\Omega_{de} \propto a^2$ with the proportionality coefficient in order of $O(1)$ by referring to the best-fit results of the model parameters. Obviously, $\Omega_{de} \ll 1$ at $a \ll 1$ in the early universe.

Substituting the results of Ω_{de} to Eq.(10), we get the equation of state of CHDE. The evolutionary trajectory of the EoS of CHDE is shown in Fig.3. It is observed that the EoS of CHDE cross over $w_{de} = -1$ from $w_{de} > -1$ to $w_{de} < -1$ during the universe expansion. The potential singularity for $w_{de} < -1$ in the future might be ceased by high order effect of gravity.

From the conservation of total energy $\dot{\rho}_{tot} + 3H(1 + w_{tot})\rho_{tot} = 0$, we have $w_{tot} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}$ by using $\rho_{tot} = 3M_p^2 H^2$. On the other hand, $w_{tot} = \frac{1}{3}\Omega_r + w_{de}\Omega_{de}$ as matter is pressureless. Combining

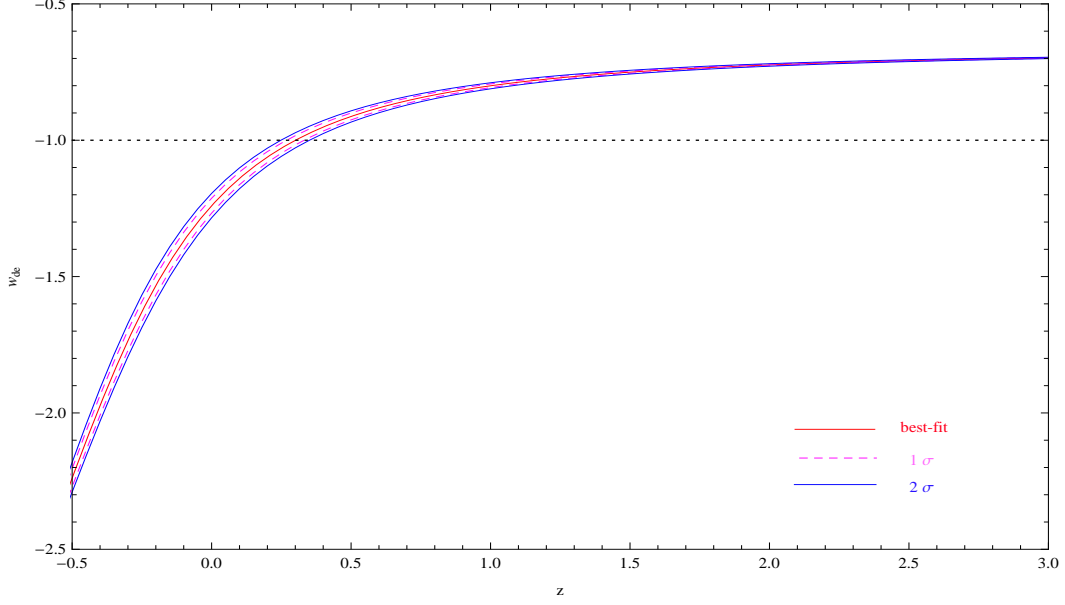


FIG. 3: Cosmic evolution of the EoS of CHDE

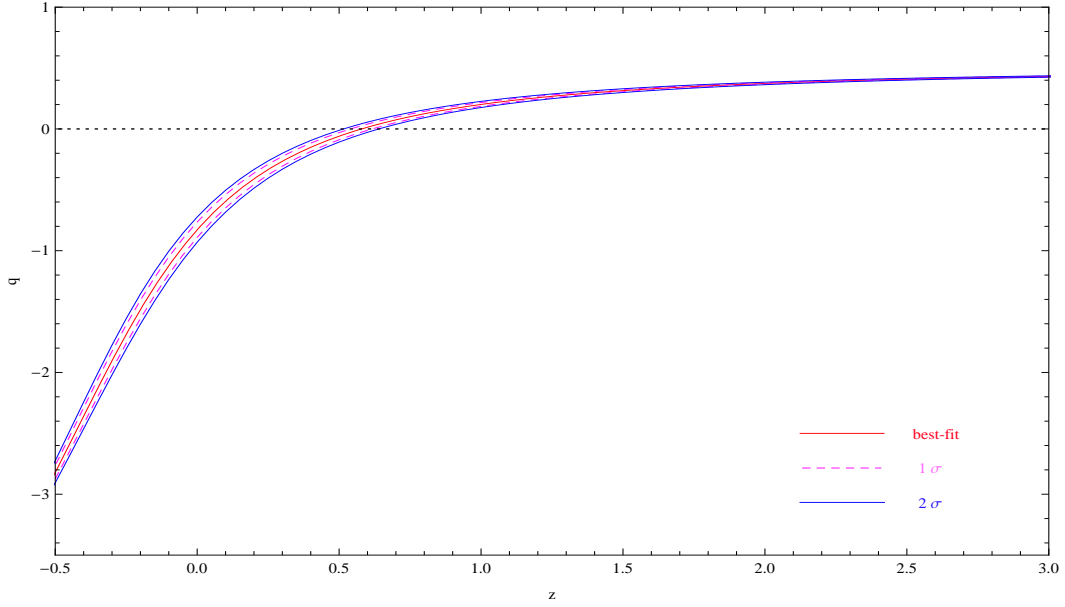


FIG. 4: Evolution of the decelerating parameter

with Eq.(12), it is not difficult to get the decelerating parameter

$$q \equiv -\frac{\ddot{a}}{aH^2} \equiv -1 - \frac{\dot{H}}{H^2} = \frac{1}{2} + \frac{1}{2} \frac{r_0(1+z)}{1+r_0(1+z)}(1 - \Omega_{de}) + \frac{3}{2} w_{de} \Omega_{de} . \quad (21)$$

Using the expression of w_{de} in Eq.(10) and substituting the results of Ω_{de} into above equation, we then yield the decelerating parameter. The evolutionary trajectory of the decelerating parameter is shown in Fig.4. It is clear that the universe does transit from the decelerated expansion $q > 0$ to

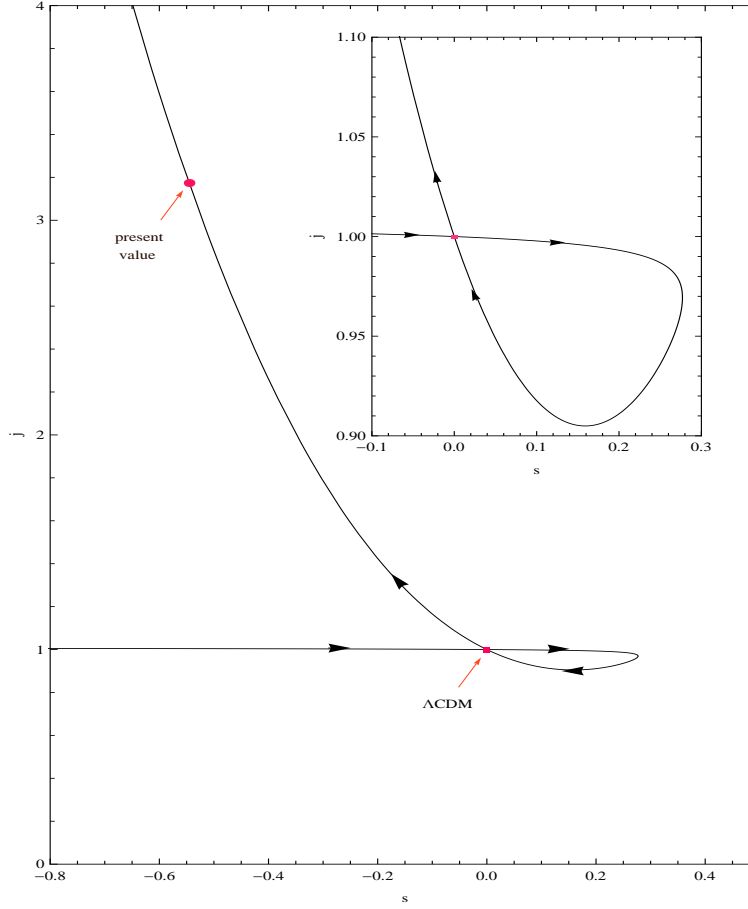


FIG. 5: The statefinder parameter $j-s$ contour evolves in redshift interval $z \in [-0.2, 15]$ (the arrow indicates the evolution from high redshift to low redshift); where model parameters take the best-fit values, i.e. $d = 0.232$ and $r_0 = 3.052 \times 10^{-4}$

the accelerated expansion $q < 0$ at the very recent epoch.

The statefinder [53, 54] are geometric parameters probing the expansion dynamics of the universe through high derivatives of the scale factor \ddot{a} and $\ddot{\ddot{a}}$. It is a natural next step beyond the Hubble parameter H depending on \dot{a} and the deceleration parameter q depending on \ddot{a} . The statefinder pair $\{j, s\}$ are defined as

$$j \equiv \frac{\ddot{\ddot{a}}}{aH^3}, \quad (22)$$

$$s \equiv \frac{j-1}{3(q-\frac{1}{2})}, \quad (23)$$

where we use j instead of r used in [54] to denote the first parameter because we have used r to denote the ratio of the fraction energy density of radiation to that of matter defined in Eq.(13).

The Λ CDM model corresponds to a fixed point $\{j, s\} = \{1, 0\}$. Departure of a given dark energy

model from this fixed point provides a good way of establishing the "distance" of this model from the Λ CDM model [54].

Using the FRW equation Eq.(11), conservation equations Eqs.(9) and the decelerating parameter Eq.(21), we can rewrite the statefinder parameters as

$$j = 1 + \frac{3}{2} \left((1+z) \frac{dw_{de}}{dz} + 3w_{de}(1+w_{de}) \right) \Omega_{de} + 2\Omega_r, \quad (24)$$

$$s = \frac{3 \left((1+z) \frac{dw_{de}}{dz} + 3w_{de}(1+w_{de}) \right) \Omega_{de} + 4\Omega_r}{9w_{de}\Omega_{de} + 3\Omega_r}. \quad (25)$$

Referring to Eq.(10) and Eq.(12), w_{de} and Ω_r can be expressed in terms of Ω_{de} . Therefore, solving the differential equation of $\Omega_{de}(z)$ and substituting the results into above equations, the statefinder parameter pair $\{j(z), s(z)\}$ can be obtained.

As the present fraction of radiation is tiny and the EoS of dark energy is negative at present epoch, the denominator of s in Eq.(25) is negative today. However, the fraction of radiation energy increases rapidly with increase of the redshift while the fraction of dark energy is tiny in the early universe. Thus, the denominator is positive in the early universe. Therefore, the statefinder parameter s might be divergent at some early time and would become meaningless. In Fig.[5], we only show the evolutionary trajectory of the statefinder parameter $j - s$ in redshift interval $z \in [-0.2, 15]$. The present statefinder of the CHDE model is significantly away from the fixed point corresponding to the Λ CDM model. Thus, statefinder may be a sensitive diagnostic to differentiate these two models.

V. CONCLUDING REMARKS

We have made the best-fit analysis on the holographic dark energy model characterized by the conformal-age-like length $L = \frac{1}{a^4(t)} \int_0^t dt' a^3(t')$ (CHDE) [41]. With the joint analysis by using the Union2 compilation of 557 supernova Ia (SNIa) data [42], the baryon acoustic oscillation results from SDSS DR7 [43] and the cosmic microwave background radiation (CMB) data from the WMAP7 [44], we have obtained the minimal χ^2 for CHDE model with $\chi_{min}^2 = 546.273$, which is slightly bigger than the one for the Λ CDM model with $\chi_{min}^2 = 544.616$. The constraints on the model parameter d at 1σ and 2σ confidence levels are found to be $d = 0.232 \pm 0.006 \pm 0.009$. Corresponding constraints on the present fractional energy density of matter and the present equation of state of CHDE are found to be $\Omega_{m0} = 0.286^{+0.019}_{-0.018} {}^{+0.032}_{-0.028}$ and $w_{de0} = -1.240^{+0.027}_{-0.027} {}^{+0.045}_{-0.044}$.

respectively. For comparison, we have also fitted the Λ CDM model to the same observational data and found that $\Omega_{m0} = 0.277^{+0.021}_{-0.019} {}^{+0.035}_{-0.031}$. It has been seen that the CHDE model leads to a slightly larger fraction of matter comparing to the Λ CDM model and the present EoS of CHDE is much less than -1 .

We have also provided a systematic analysis on the CHDE model for its cosmic evolutions of the fractional energy density of dark energy, the EoS of dark energy, the deceleration parameter and statefinder. From the evolutionary trajectory of Ω_{de} , it has been found that the fraction of dark energy decreases rapidly with the increase of the redshift and becomes tiny in the early universe. Thus, the model is consistent with the primordial nucleosynthesis (BBN) [55]. The EoS of CHDE has been shown to cross from $w_{de} > -1$ to $w_{de} < -1$ during the universe expansion. The evolution of the deceleration parameter has indicated that the universe transits from decelerated expansion $q > 0$ to accelerated expansion $q < 0$ recently. It has been noticed that the statefinder may provide a sensitive diagnostic to differentiate the CHDE model with the Λ CDM model.

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Appendix A: Analysis method for the observational data

In this appendix, we explain the methods for the elaboration of observational data from the Union2 compilation of 557 supernova Ia (SNIa) data [42], the baryon acoustic oscillation (BAO) results from the Sloan Digital Sky Survey data release 7 (SDS DR7) [43] and the cosmic microwave background radiation (CMB) data from the 7-yr Wilkinson Microwave Anisotropy Probe (WMAP7) [44]. For the three independent observations, the likelihood function of a joint analysis is given by

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\text{SN}} \times \mathcal{L}_{\text{BAO}} \times \mathcal{L}_{\text{CMB}} \\ &= \exp[-(\tilde{\chi}_{\text{SN}}^2 + \chi_{\text{BAO}}^2 + \chi_{\text{CMB}}^2)/2] . \end{aligned} \quad (\text{A1})$$

The model parameters yielding a maximal \mathcal{L} , thus a minimal $\chi^2 = \tilde{\chi}_{\text{SN}}^2 + \chi_{\text{BAO}}^2 + \chi_{\text{CMB}}^2$, will be favored by the observations. In the following, we present the calculation for the various χ_i^2 of each observational data set, and mainly adopt the analysis method described in [45–47].

a. Type Ia Supernovae (SN Ia)

The SN Ia observations give the information on the luminosity distance D_L . The distance modulus is theoretically defined as the function of the redshift z

$$\mu_{\text{th}}(z_i) \equiv 5 \log_{10} D_L(z_i) + \mu_0, \quad (\text{A2})$$

with $\mu_0 \equiv 42.38 - 5 \log_{10} h$ and $h \equiv H_0/100/[\text{km sec}^{-1} \text{Mpc}^{-1}]$. The Hubble-free luminosity distance has the following form for the flat universe

$$D_L(z) = (1+z) \int_0^z \frac{dz'}{E(z')}, \quad (\text{A3})$$

with $E(z) \equiv H(z)/H_0$.

The χ^2 for the SNIa data is given by

$$\chi_{\text{SN}}^2 = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma_i^2}, \quad (\text{A4})$$

where $\mu_{\text{obs}}(z_i)$ and σ_i are the observed quantity and the corresponding 1σ error of distance modulus for each supernova, respectively. Adopting the approach in [48], the χ_{SN}^2 with respect to μ_0 can be expanded as

$$\chi_{\text{SN}}^2 = A - 2\mu_0 B + \mu_0^2 C, \quad (\text{A5})$$

with

$$A = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0)]^2}{\sigma_i^2}, \quad (\text{A6})$$

$$B = \sum_i \frac{\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0)}{\sigma_i^2}, \quad (\text{A7})$$

$$C = \sum_i \frac{1}{\sigma_i^2}. \quad (\text{A8})$$

It is easy to check that the minimum of χ_{SN}^2 with respect to μ_0 is given by

$$\tilde{\chi}_{\text{SN}}^2 = A - \frac{B^2}{C}. \quad (\text{A9})$$

which is applied in our best-fit analysis to the χ^2 minimization by using the Supernova Cosmology Project (SCP) Union2 compilation, which contains 557 supernovae [42] with the range of the redshift $z = 0.015 - 1.4$.

b. Baryon Acoustic Oscillations (BAO)

The distance ratio $d_z \equiv r_s(z_d)/D_V(z)$ is measured by BAO observations, with r_s the comoving sound horizon, D_V the volume-averaged distance and z_d the redshift of the baryon drag epoch [43].

The comoving sound horizon $r_s(z)$ is given by

$$r_s(z) = \frac{1}{\sqrt{3}} \int_0^{1/(1+z)} \frac{da}{a^2 H(a) \sqrt{1 + (3\Omega_{b0}/4\Omega_{\gamma0})a}}, \quad (\text{A10})$$

with Ω_{b0} and $\Omega_{\gamma0}$ corresponding to the present baryon and photon density parameters. We take the best-fit values: $\Omega_{b0} = 0.02253h^{-2}$ and $\Omega_{\gamma0} = 2.469 \times 10^{-5}h^{-2}$ (for $T_{CMB} = 2.725$ K) obtained by the 7-yr WMAP observations [44].

The volume-averaged distance $D_V(z)$ is defined as [49]

$$D_V(z) \equiv \left[(1+z)^2 D_A^2(z) \frac{z}{H(z)} \right]^{1/3}, \quad (\text{A11})$$

with $D_A(z)$ the proper angular diameter distance, which is defined for the flat universe by

$$D_A(z) \equiv \frac{1}{1+z} \int_0^z \frac{dz'}{H(z')}. \quad (\text{A12})$$

and z_d is given by [50]

$$z_d = \frac{1291(\Omega_{m0}h^2)^{0.251}}{1 + 0.659(\Omega_{m0}h^2)^{0.828}} \left[1 + b_1(\Omega_{b0}h^2)^{b_2} \right], \quad (\text{A13})$$

with

$$b_1 = 0.313(\Omega_{m0}h^2)^{-0.419} \left[1 + 0.607(\Omega_{m0}h^2)^{0.674} \right], \quad b_2 = 0.238(\Omega_{m0}h^2)^{0.223}. \quad (\text{A14})$$

From the Two-Degree Field Galaxy Redshift Survey (2dFGRS) and the Sloan Digital Sky Survey Data Release 7 (SDSS DR7) [43], we have the values for the distance ratio $d_{0.2}^{\text{obs}} = 0.1905$ and $d_{0.35}^{\text{obs}} = 0.1097$ corresponding to the two redshifts $z = 0.2$ and $z = 0.35$. The χ^2 of the BAO data is given by:

$$\chi_{\text{BAO}}^2 = Y^T C_{\text{BAO}}^{-1} Y, \quad (\text{A15})$$

with $Y = \left(d_{0.2}^{\text{th}} - d_{0.2}^{\text{obs}}, d_{0.35}^{\text{th}} - d_{0.35}^{\text{obs}} \right)^T$, and the inverse covariance matrix

$$C_{\text{BAO}}^{-1} = \begin{pmatrix} 30124 & -17227 \\ -17227 & 86977 \end{pmatrix}. \quad (\text{A16})$$

c. Cosmic Microwave Background (CMB)

For the CMB data, we shall use the acoustic scale l_A , the shift parameter R and the redshift of the decoupling epoch of photons z_* . They are defined as [51]

$$l_A(z_*) \equiv (1 + z_*) \frac{\pi D_A(z_*)}{r_s(z_*)}, \quad (\text{A17})$$

$$R(z_*) \equiv \sqrt{\Omega_{m0}} H_0 (1 + z_*) D_A(z_*), \quad (\text{A18})$$

with r_s given in Eq.(A10). The redshift of the decoupling epoch z_* is given by [52]

$$z_* = 1048[1 + 0.00124(\Omega_{b0}h^2)^{-0.738}][1 + g_1(\Omega_{m0}h^2)^{g_2}], \quad (\text{A19})$$

with

$$g_1 = \frac{0.0783(\Omega_{b0}h^2)^{-0.238}}{1 + 39.5(\Omega_{b0}h^2)^{0.763}}, \quad g_2 = \frac{0.560}{1 + 21.1(\Omega_{b0}h^2)^{1.81}}. \quad (\text{A20})$$

From WMAP7 observations [44], we have the corresponding CMB data $l_A^{\text{obs}}(z_*) = 302.09$, $R^{\text{obs}}(z_*) = 1.725$ and $z_*^{\text{obs}} = 1091.3$. The resulting χ^2 is

$$\chi_{\text{CMB}}^2 = X^T C_{\text{CMB}}^{-1} X, \quad (\text{A21})$$

with $X = \left(l_A^{\text{th}}(z_*) - l_A^{\text{obs}}(z_*), \quad R^{\text{th}}(z_*) - R^{\text{obs}}(z_*), \quad z_*^{\text{th}} - z_*^{\text{obs}} \right)^T$, and the inverse covariance matrix

$$C_{\text{CMB}}^{-1} = \begin{pmatrix} 2.305 & 29.698 & -1.333 \\ 29.698 & 6825.27 & -113.180 \\ -1.333 & -113.180 & 3.414 \end{pmatrix}. \quad (\text{A22})$$

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